Math 113. Sample Final Exam

Let F be a finite field of order 49, R be the ring $\mathbf{Z}/144\mathbf{Z}$ of residues modulo 144.

1. Show that the multiplicative groups $F^\times$ and $R^\times$ have the same order N and find this N.

2. Classify all abelian groups of order N up to isomorphism. Which of these groups are isomorphic to $F^\times$?

3. Prove that the ring R is isomorphic to the direct product of the rings $\mathbf{Z}/9\mathbf{Z}$ and $\mathbf{Z}/16\mathbf{Z}$.

4. Use **3** in order to find the place of $R^\times$ in your classification.

5. Construct explicitly a field F of order 49 and point out a generator of the group $F^\times$. How many choices of such a generator are available in the group $F^\times$?

Find all those n for which the permutation group S_n contains:

6. a subgroup of order 60;

7. a cyclic subgroup of order 60.

8. For any field F, find the greatest common divisor in $F[x]$ of the polynomials $x^m - 1$ and $x^n - 1$.

9. Trisect the angle $\pi/7$ using only straightedge and compass.

10. To commemorate two centuries of Gauss' *Disquisitiones Arithmeticae* the Institute of Mathematical History is selling necklaces priced $62.50 each and consisting of 17 identically shaped symmetrical black or white beads moving freely on a circular band. Find the price the Institute of Historical Mathematics will have to pay for a complete collection of such necklaces.